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CASE STUDIES IN CONCEPT SELECTION USING S-PARETO FRONTIERS

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ABSTRACT

In recent publications, we introduced a new multiobjective optimization-based concept selection framework for engineering design. The new concept selection framework, which is briefly described in this paper, capitalizes on the efficiency and effectiveness of optimization to rapidly compare disparate design concepts by characterizing the concepts' strengths and weaknesses within the multiobjective design space. Specifically, the new framework is used to identify the tradeoffs between competing design objectives *and* those between disparate design concepts. This paper presents three diverse case studies that illustrate the usefulness of the new concept selection framework. The first case study considers the design of a simple truss structure, while the second examines the design of a complex system – a rigidified inflatable structure. In the final case study, the design of a consumer product is considered. In each case study, the new framework forms a sound basis for the decision-making process.

INTRODUCTION

The engineering design process generally requires the designer to resolve various competing design objectives. To identify an optimal design, competing objectives are typically considered simultaneously. A powerful tool for resolving such objectives, in a computational setting, is multiobjective optimization. A particular class of optimal solutions to the multiobjective optimization problem is referred to as the Pareto optimal set, or Pareto frontier. By definition, Pareto solutions are considered optimal because there are no other designs that out perform them in all objectives [1].

Recently, there have been increased efforts to develop approaches for conceptual design that capitalize on the benefits of multiobjective optimization. The primary motivation for such a thrust is that there exists significant design freedom early in the design process; thereby allowing for greater potential benefits, and significant positive impact on the success of the final design [2, 3].

In a recent publication [2], we presented a new Pareto frontier-based approach to concept selection in conceptual engineering design. Under the new framework, disparate design concepts are evaluated using a so-called *s-Pareto frontier*; this frontier originates from the Pareto frontiers of various disparate concepts, and *is* the Pareto frontier for the *set* of concepts. Similar to other Pareto frontiers, the s-Pareto frontier can be used to characterize the tradeoffs between design objectives. *However, unlike other Pareto frontiers, the s-Pareto frontier can be used to characterize the tradeoffs between disparate design concepts.* This property of the s-Pareto frontier is what makes it extremely useful for decision-making in conceptual design.

The significance of the s-Pareto frontier is that it makes it possible to use optimization to explore the design space in the early phases of design – as it pertains to more than one concept – and while decisions have a significant impact on the design success. Important developments to the s-Pareto frontier-based concept selection framework include the consideration of uncertainty, the quantification of concept goodness, and methods for visualizing *n*-dimensional s-Pareto frontiers [4].

In this paper, we present three diverse case studies that illustrate the general usefulness of the s-Pareto frontier. The first case study considers the design of a simple truss structure, while the second examines the design of a complex system – a rigidified inflatable structure. In the third case study, the design of a consumer product is considered. In each case study, the new framework forms a sound basis for the decision-

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making process.

In the following section, we provide a brief description of the s-Pareto frontier and its meaningful role in concept selection. Following the s-Pareto frontier description, three case studies are presented. Finally, concluding remarks are made in the last section.

S-PARETO FRONTIER BASED CONCEPT SELECTION

In this section, we briefly describe the meaning of *s-Pareto frontier*, and show how it can play an important role in concept selection. Before describing the s-Pareto frontier, we clarify our use of two terms used in this paper; *Design Concept* and *Design Alternative*. A design concept is an idea that has evolved to the point where there is a parametric model (however rudimentary) that represents the performance of the family of specific designs that belong to that concept's definition. A design alternative, on the other hand, is a unique design resulting from specific parameter values used in the parametric model of a concept.

Selecting a design concept is generally the focus of conceptual design, while selecting a design alternative is typically the focus of detailed design. We assume that each design concept has its own Pareto frontier (comprising numerous design alternatives), which is defined by the concept's parametric model. The heavy line of Fig. 1(a) represents the Pareto frontier for a bi-objective minimization problem, where μ_1 and μ_2 are the design objectives, and the shaded region represents the feasible space for a given concept. Each solution comprising the frontier is a design alternative.

To introduce the notion of an s-Pareto frontier, we consider Fig. 1(b) and the accompanying design space for the design objectives μ_1 and μ_2. Candidate design concepts are represented by the shaded areas (feasible regions) labeled A, B, and C (referring to concepts A, B, and C, respectively). Each concept is unique in that it has its own parametric model that defines the space it occupies. The model quantifies the performance of a family of design alternatives, which belong to that concept's definition. The set of potentially optimal alternatives comprise the individual Pareto frontiers (black curves) of each concept.

The s-Pareto frontier originates from the Pareto frontiers of one or more of the design concepts and is the Pareto frontier for the set of concepts. The s-Pareto frontier is illustrated by the heavy line in Fig. 1(c). Each solution comprising the s-Pareto frontier is said to be *s-Pareto optimal*, which means there are no other designs – from the same or any other concept – for which all objectives are better. Formally, we define s-Pareto optimality as follows.

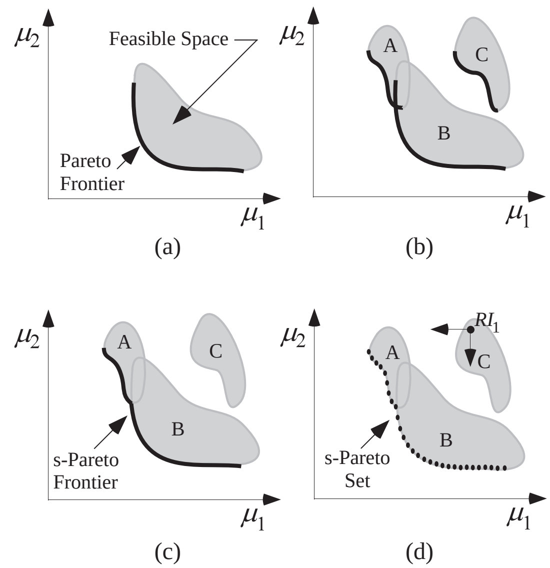


Figure 1. Basic Introduction to the s-Pareto frontier: (a) Pareto frontier for one concept, (b) Pareto frontiers for three concepts, (c) s-Pareto frontier, (d) discrete representation of s-Pareto frontier.

s-Pareto Optimality: A design alternative μ^{s*} is s-Pareto optimal if there does not exist another design alternative μ^k in the feasible design space of concept k such that $\mu_i^k \le \mu_i^{s*}$ for all $i \in \{1,2,\ldots,n\}$ and all concepts k, where $k \in \{1,2,\ldots,n_c\}$; and $\mu_j^k < \mu_j^{s*}$ for at least one j, $j \in \{1,2,\ldots,n\}$ for any concept k, $k \in \{1,2,\ldots,n_c\}$. The number of design concepts is denoted by n_c, and the number of design objectives is denoted as n.

The "s" in *s-Pareto frontier* indicates that the Pareto frontier is for the *set* of concepts. Similar to other Pareto frontiers, the s-Pareto frontier can be used to determine the tradeoffs between design objectives. However, unlike other Pareto frontiers, the s-Pareto frontier can be used to characterize the tradeoffs between design concepts. For example, the s-Pareto frontier in Fig. 1(c) shows that concept C is inferior to concepts A and B, both of which are not dominated. Furthermore, it can be seen that concept A is superior to concept B when low values of μ_1 are preferred, and that concept B is superior for low values of μ_2. Importantly, the point at which one concept becomes superior to another – for any objective – can also be identified by examining the s-Pareto frontier. We make the important observation that unlike decision matrix based methods where concept selection is based on a single performance number for the objectives, the s-Pareto approach accounts for the objectives behaviors over ranges.

Various approaches may be used to obtain a set of points that discretely represent the *s-Pareto frontier*. This set, referred to as the *s-Pareto set*, is illustrated in Fig. 1(d). Approaches for obtaining the s-Pareto set include the following: (i) Obtaining Pareto sets for each concept (using any Pareto set generator), and filtering out any solution that does not satisfy the definition of s-Pareto optimality. (ii) Directly obtaining the s-Pareto set using a single optimization problem statement. The latter is discussed in detail in [2]. The method presented in [2] yields a set of evenly distributed points along the s-Pareto frontier, not unlike the distribution of points illustrated in Fig. 1(d). We note that by even distribution, we mean that no one part of the Pareto frontier is over or under represented in the Pareto set.

Once the s-Pareto set has been obtained, the process of using it to identify the dominance disposition of the candidate concepts can begin. A useful two-phase approach for doing this is to (i) define a *region of interest*, and (ii) quantify the goodness of each concept within that region. Below, we provide a brief description of these two important phases. More comprehensive details are provided in Mattson and Messac [4].

The designer evaluates the goodness of the concepts within a particular region of the design space that is of interest to him or her. We call this a *Region of Interest*. The region southwest of the point RI_1 in Fig. 1(d) is an example of a region of interest. By exploring various regions of interest, the designer can collect information about the design space (i.e., which concepts occupy which parts of the design space); this information is then used to identify the concept or concepts that merit further development.

In evaluating concept goodness, we examine Pareto frontier surface areas and assume that concepts whose Pareto frontiers have larger surface areas potentially offer more design *flexibility* than those with smaller Pareto surfaces. More flexible concepts are assumed to be preferred because they provide more design freedom for detailed design.

As described in Mattson and Messac [4], the goodness of each concept is quantified by determining the intersection of a concept's Pareto frontier with the s-Pareto frontier. Mathematically, the goodness of the i-th concept is expressed as

$$\Gamma_i = \frac{\int_{S_p \cap S_{pi}} dS_p}{\int_{S_p} dS_p} \quad (1)$$

where S_p is the s-Pareto frontier, and S_{pi} is the Pareto frontier for the i-th concept. The numerator and denominator in Eq. (1) are n dimensional integrals. Equation (1) denotes the fraction of the s-Pareto frontier that originates from the i-th concept. An approximation of this goodness measure is now provided for the discrete domain. Given a set of evenly distributed points along the s-Pareto frontier, this measure of goodness can be expressed as $\Gamma_i \approx n_{si}/n_s$ where n_s is the total number of s-Pareto solutions and n_{si} is the number of s-Pareto solutions originating from the i-th concept. For a comprehensive description of the s-Pareto frontier-based concept selection framework see Mattson and Messac [4, 2].

In the following section we examine the usefulness of the s-Pareto frontier for concept selection. Specifically, we examine three unique case studies – each illustrating a different and important aspect of the s-Pareto frontier-based concept selection framework.

CASE STUDIES IN CONCEPT SELECTION

In this section, we describe three case studies that illustrate the usefulness of the s-Pareto frontier in conceptual design and decision making. We now provide a brief description of each of the three concept selection problems, and focus primarily on the results obtained and their meaning. Importantly, we refer to our additional publications on the topic [4, 2] for important supplementary details regarding these cases.

The three case studies, presented in this paper, are used to examine the applicability of the s-Pareto frontier-based concept selection framework – each case study illustrates a different and important aspect of the framework. The first case study is the design of a simple tractable structure. Its purpose is to illustrate the basic components of the new concept selection framework through the design of a simple structure, where results can be easily reproduced by others. The second case study is the conceptual design of a complex system – a rigidified inflatable structure. This case study is used to show that the s-Pareto frontier-based concept selection framework can be used to evaluate complex systems. The third case study is the design of a compliant bicycle derailleur. The purpose of this case study is to show how the new framework

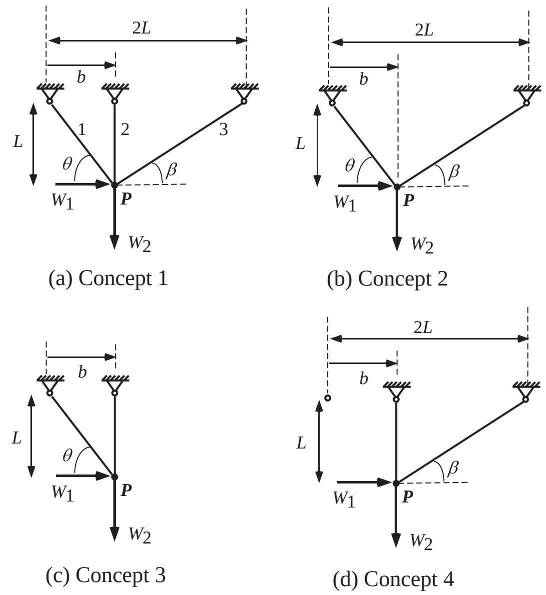


Figure 2. Candidate Truss Concepts.

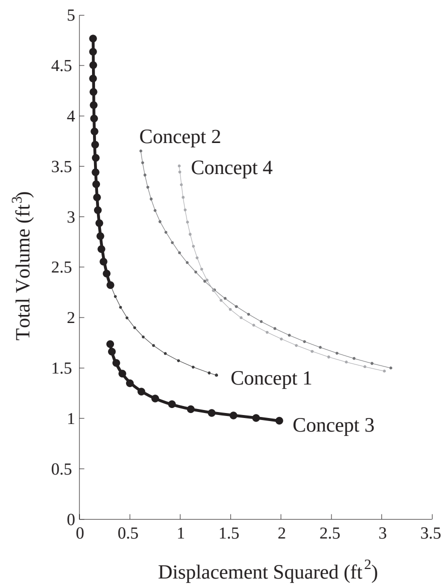


Figure 3. Pareto frontier and s-Pareto frontier (heavy curve) for simple structural case.

can be used in conjunction with other decision making approaches such as feasibility judgment and prototype testing.

Case Study 1: Simple Structural Design

The design of a simple truss structure is examined in this case study. This case shows that the s-Pareto frontier can be used to quickly eliminate dominated truss concepts, and identify the tradeoffs between the non-dominated concepts. Importantly, the s-Pareto frontier is used to characterize the multiobjective design space for this simple example. We begin this case study by describing the basic truss design problem, followed by a description of the candidate truss concepts. The basic optimization problem statement is then given and the results are presented and discussed.

Truss Design Problem: Design a truss structure that minimizes the nodal deflection at a critical predetermined node, P, and minimizes the total structural volume, subject to the normal stress and beam cross-sectional areas being within acceptable levels.

Candidate Truss Concepts: Given the basic description of the truss design problem above, we generate four truss concepts using traditional concept generation methods. The generated concepts are shown in Fig. 2. Each of the truss concepts is subject to two applied loads; a horizontal load, W_1, and a vertical load, W_2. Both loads act on the node P. The dimension b defines the horizontal distance between the left-most pin-joint and the node P. Each of these candidate concepts is of height L, and none is greater than $2L$ in length. Concept 1 is a three-bar truss structure, while Concepts 2, 3, and 4 are two bar structures. The vertical member of Concepts 1, 3, and 4 is required to remain vertical, while the angle of the other members is determined by the parameters θ or β.

In the context of the design process, our objective at this point is to select the most desirable truss concept or concepts. The selected concepts will go forward into the detailed design phase. The s-Pareto frontier is used to identify the strengths and weaknesses of the candidate concepts.

Optimization Problem Statement, Study 1: The optimization problem statement for the simple structural design problem is given as follows

$$\min_{a,b} \quad [\mu_1(a,b) \quad \mu_2(a,b)]^T \tag{2}$$

subject to

$$S_i \leq S_{\max} \qquad i = 1,2,3 \tag{3}$$

$$0.8\,\text{in}^2 \leq a_i \leq 3\,\text{in}^2 \quad i = 1,2,3 \tag{4}$$

$$L/2 \leq b \leq 3L/2 \tag{5}$$

where μ_1 is the squared nodal deflection at node P and μ_2 is the structural volume. The stress in each bar must be lower than the maximum allowable stress, as indicated by Eq. (3), where the bar on the left (Fig. 2(a)) is bar 1, the vertical bar is bar 2, and the bar on the right is bar 3. The cross sectional area of each bar, a_i, is limited as described by Eq. (4), and the horizontal location of node P, b, is also constrained by Eq. (5). The fixed parameters for this problem are defined as follows. Young's Modulus, E, is 29x10^3 ksi; truss dimension L is 60 ft; the maximum allowable stress, $S_{\max}$, is 550 ksi; and the loads W_1 and W_2 are 100 kips and 1,000 kips, respectively.

Results and Discussion: The Pareto frontier for each of the truss concepts is obtained (for the basic optimization problem statement Eqs. (2) – (5)). Figure 3 shows these frontiers. The Pareto frontier for each concept is shown as a solid curve. The solid points are each concept's Pareto solutions, which were obtained using the Normal Constraint method [5] and the developments presented in [2]. An s-Pareto set is obtained for the three truss concepts, which is shown as a set of large points in Fig. 3, and the s-Pareto frontier is shown as the heavy curve.

Figure 3 explicitly shows the meaning of the s-Pareto frontier, which captures the tradeoff properties between concepts *and* the tradeoffs between objectives; and *is* the Pareto frontier for the set of concepts. For this bi-objective case, we explore the design space by visually examining the two dimensional s-Pareto frontier. Doing so, we can specifically conclude that (i) Concept 1 is superior if low nodal displacement is desired, (ii) Concept 2 is inferior and can be eliminated, (iii) Concept 3 is superior if low structural volume is desired, and (iv) Concept 4 is also dominated and can be eliminated. Importantly, when a compromise between displacement and volume is desired, the designer can explore various regions of interest, and use the measure of concept goodness (see Eq. (1)) to draw conclusions about the tradeoffs between the non-dominated concepts.

Case Study 2: Complex Structural Design

This section presents a case study where three different materials are evaluated for use in a large structural system. Ultimately, the material(s) that performs favorably is identified. The purpose of this case study is to illustrate two important aspects of the s-Pareto frontier based concept selection framework. The first being that the s-Pareto frontier can be used for multiobjective decision making in general, such as the material selection problem presented here. The second point, illustrated in this case study, is that when complex, expensive, models characterize the concepts under evaluation, we must judiciously generate a minimal number of s-Pareto solutions in order to make the needed decision.

Table 1. Material properties for RIS case study

Mat'l Class	Modulus of Elasticity (GPa)	Tensile Strength (MPa)	Density (kg/m^3)	Cost ($/kg)
1	3.0	30.0	1400	2.50
2	10.0	100.0	1900	11.00
3	30.0	300.0	2100	22.00

We consider the conceptual design of a Rigidified Inflatable Structure (RIS). As described in Messac et al. [6], rigidified inflatable structures are thin flexible membranes that rigidify after pneumatically deploying. The resulting structure is a thin-shell structure that is rigid and capable of supporting loads. In a conceptual design study for RIS-based residential housing, Messac et al. [6] developed a RIS testbed that was used to examine the feasibility of three candidate RIS materials. In the present case study, we use the same RIS testbed to obtain the s-Pareto frontier for the three materials. Doing so allows us to identify ranges of objectives behaviors for each material type. This marked departure from previous RIS studies, is facilitated by the use of the s-Pareto frontier.

RIS Design Problem: Select a RIS material that minimizes structural deflection, and minimizes material cost. Also, ensure that the stress does not exceed acceptable limits.

Candidate RIS Concepts: Three design options that correspond to three different material types are considered. The three materials considered are listed in Table 1 along with their material properties. The materials in Class 1 are non-reinforced polymers; the materials in Class 2 are polymers lightly reinforced with randomly oriented, discontinuous, E-glass fibers; and the materials in Class 3 are polymers reinforced with uniformly oriented, continuous, E-glass fibers.

Our primary objective, from a design standpoint, is to select the most appropriate RIS material from the set of candidates. We use the s-Pareto frontier to characterize the candidate materials.

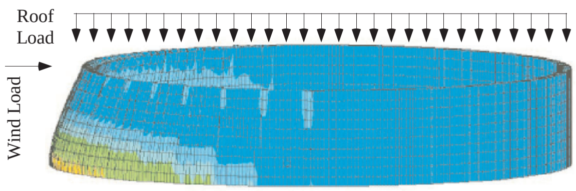


Figure 4. RIS exaggerated deflection plot, with applied loads.

Optimization Problem Statement: The optimization problem statement for the RIS case study is presented as follows:

$$\min_{t_1,t_2,t_3} \quad [c \quad \delta_y]^T \tag{6}$$

subject to

$$2\sigma \leq \sigma_{\text{tensile}} \tag{7}$$

$$\delta_x, \delta_y \leq 6 \text{ cm} \tag{8}$$

$$t_{\min} \leq t_1, t_2, t_3 \leq t_{\max} \tag{9}$$

where c is the total material cost, and δ_y is the maximum deflection in the direction of the wind load. Equation (7) constrains the stresses of all elements to be less than half the material tensile strength, σ_{tensile}. Equation (8) constrains the deflections of all nodes in the x and y directions to be less than the maximum deflection enforced by typical building codes. The last constraint enforces the upper and lower bounds of the allowable membrane thicknesses.

Results and Discussion: The complexity of the structure discussed above requires the use of finite element modeling and analysis. A finite element program called Genesis (Vanderplaats R&D, [7]) is used for the modeling, analysis, and optimization. Important modeling issues are discussed in Messac et al. [6].

To evaluate the viability of the three materials, we take a strategic approach to identify the s-Pareto solutions. Such an approach is critical for this problem, because obtaining one Pareto solution required approximately 1 hour of computation time. We started by obtaining the end points of the Pareto frontier for each material. These are shown in Fig. 5. From this information, it became clear that Material 1 would offer the lowest cost solution, and Material 3 would offer the lowest deflection design. It was unclear, however, how the tradeoffs between the materials would be characterized away from the frontier end points. We strategically chose the regions in which we wished to search

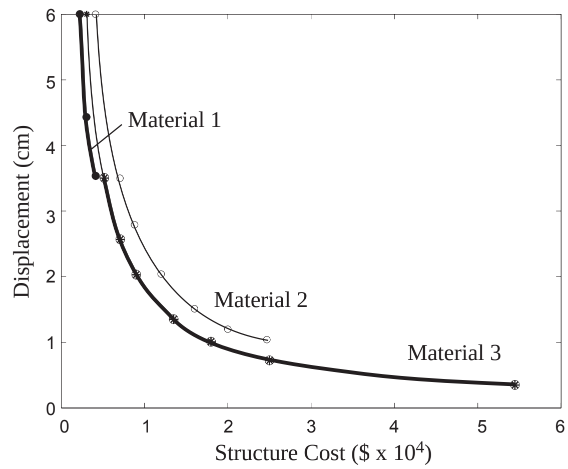


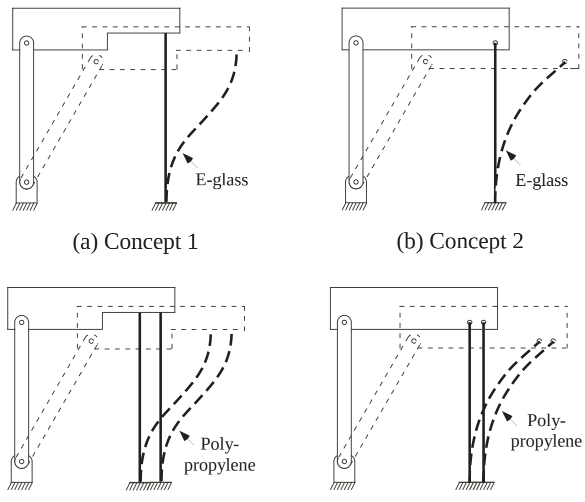
Figure 5. Judiciously generated Pareto solutions, and the approximate s-Pareto frontier (heavy curve) for the complex structural case.

for optimal solutions. For example, we applied additional constraints that allowed us to identify the cost values for Materials 2 and 3 at the lowest deflection value of Material 1. Finally after identifying 18 Pareto solutions, we were able to feel confident that Material 2 is dominated by Materials 1 and 3. Therefore as a result of this study, Material 2 can be removed from the list of candidate RIS materials, and Materials 1 or 3 may be further developed depending on which material the designer finds more desirable.

Case Study 3: Consumer Product Design

In this section, we consider the design of a small consumer product – a bicycle derailleur. The purpose of this case study is to illustrate how the s-Pareto frontier-based concept selection framework fits into the conceptual design process. We begin this section with the basic derailleur design problem, followed by a description of the candidate derailleur concepts. We then provide the basic optimization problem statement; the results and related discussion are then presented.

Derailleur Design Problem: Design a compliant bicycle derailleur that is lighter than a rigid-body derailleur of similar force-deflection characteristics. Consider the Shimano Deore XT derailleur to be the benchmark design. We note that a compliant derailleur is a four-bar mechanism that gains some or all of its motion through the large deflection of one or more of its links.



(a) Concept 1 (b) Concept 2

(c) Concept 3 (d) Concept 4

Figure 6. Candidate Derailleur Concepts

Candidate Derailleur Concepts: As the initial steps in the conceptual design process, various compliant derailleur concept were generated. Specifically, twenty-eight possible design configurations for the compliant derailleur were identified using the Pseudo-Rigid-Body Model and type synthesis [8, 9]. Using feasibility judgement, we reduce the set of twenty-eight designs down to two promising configurations. One of the configurations is shown in Fig. 6(b). This configuration has one compliant member (shown as a thin line), which is fixed at one end and pinned at the other. A second configuration is shown in Fig. 6(a). This configuration also has one compliant member, however, both ends are fixed. The compliant link for Concepts 1 and 2 is an E-glass composite, while Concepts 3 and 4 use multiple strips of polypropylene for the compliant members. Concept 3 has 14 polypropylene strips (only two are shown in the figure).

The overall goal is to identify the most promising derailleur concepts, based on the following design objectives; (i) minimize the mass of the compliant member, (ii) maximize the force required to deflect the mechanism (within reasonable bounds), and (iii) maximize the safety factor on bending stress in the compliant members.

Optimization Problem Statement: The basic optimization problem statement for the compliant bicycle derailleur design is given as follows

$$\min_{b,h} \quad [\mu_1 \quad -\mu_2 \quad -\mu_3]^T \tag{10}$$

subject to

$$1.1 \leq \mu_3 \tag{11}$$

$$5 \text{ lb} \leq \mu_2 \leq 12 \text{ lb} \tag{12}$$

$$0.01 \text{ in} \leq b \leq 1 \text{ in} \tag{13}$$

$$h_{\min} \leq h \leq h_{\max} \tag{14}$$

where μ_1 is the mass of the compliant link, μ_2 is the output force, μ_3 is the safety factor on bending stress for the compliant link, b is the width of the compliant link, and h is its thickness. Importantly, $h_{\min}$ is 0.01 inches for Concepts 1 and 2, and 0.005 inches for Concepts 3 and 4. Additionally, $h_{\max}$ is 0.05 inches for all concepts. We now obtain the s-Pareto frontier, and use it to characterize the goodness of the derailleur concepts.

Results and Discussion: Figure 7 shows the s-Pareto set for the derailleur concepts. Immediately, it can be seen that Concepts 3 and 4 do not comprise the s-Pareto set (they are dominated concepts). We make the important note that we have identified these concepts as dominated only after we have explored the objectives behaviors over ranges – an approach that is markedly different from traditional concept selection approaches where objectives ranges are not typically considered (e.g., decision matrices).

We now explore the s-Pareto frontier to characterize the strengths and weaknesses of these two remaining concepts (Concepts 1 and 2). We note that in the previous case studies, we identified the dominance disposition of the concepts by visually examining the s-Pareto frontier. This visual examination may not always be possible, as is the case for this three dimensional design problem. We use, instead, an interactive s-Pareto frontier exploration tool, developed in Mattson and Messac [4], to identify the dominance disposition of each concepts. With each exploration, the measure of concept goodness is evaluated (see Eq. (1)). Table 2 shows the results of the exploration for this case study. Results from five explorations (five regions of interest) are provided in the table; one exploration per column. The first three rows of the table indicate specific values for each design metric. Only regions that are better than these specified values are explored – analogous to the region southwest of the point RI_1 in Fig. 1d. The measure of goodness for the i-th concept

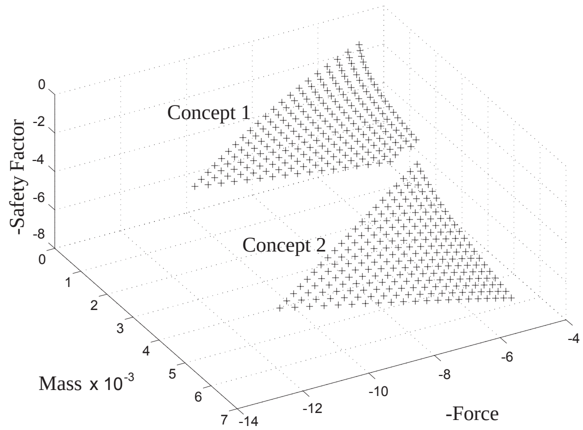


Figure 7. An s-Pareto set for the derailleur concepts.

Table 2. Derailleur Concept Exploration Results

	RI_1	RI_2	RI_3	RI_4	RI_5
μ_1	0.0064	0.0003	0.0064	0.0064	0.002
μ_2	5.000	5.000	12.00	5.000	10.026
μ_3	1.189	1.189	1.189	7.3968	1.1896
Γ_1	0.49	1	0.50	0	1
Γ_2	0.51	0	0.50	1	0
Γ_3	0	0	0	0	0
Γ_4	0	0	0	0	0

is then provided in the subsequent rows of the table.

As a result of the exploration, it can be seen that Concepts 3 and 4 are dominated, which is consistent with what we concluded by visually examining the s-Pareto set. It can also be seen that Concepts 1 and 2 are partially dominant. The measure of goodness is used to determine that approximately 49% of the s-Pareto frontier originates from Concept 1, and that approximately 51% originates from Concept 2. Further exploring the s-Pareto frontier, we see that Concept 1 is superior in maximizing the force and minimizing the mass (Column 5 of Table 2), while Concept 2 is superior for maximizing the safety factor (Column 4 of Table 2). A recent publication by Mattson et al. [9], reports on a prototyped version of Concept 1.

CONCLUDING REMARKS

In this paper, we have examined the applicability of a new concept selection framework for engineering design – the s-Pareto frontier-based concept selection framework. Three case studies were examined and show that the new framework capitalizes on the power of computational optimization by using it to evaluate disparate concepts early in the design process – before design freedoms have been significantly reduced. Specifically, in each of the cases examined in this paper, the s-Pareto frontier is used to eliminate dominated concepts, and characterize the relative strengths and the weaknesses of the non-dominated concepts.

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REFERENCES

[1] K. M. Miettinen. *Nonlinear Multiobjective Optimization*. International Series in Operations Research & Management Science. Kluwer Academic Publishers, 1999.

[2] C. A. Mattson and A. Messac. Concept selection using s-Pareto frontiers. *AIAA Journal*, 41(6):1190–1198, 2003.

[3] T. W. Simpson, D. Rosen, J. K. Allen, and F. Mistree. Metrics for assessing design freedom and information certainty in the early stages of design. *ASME Journal of Mechanical Design*, 120:628–635, 1998.

[4] C. A. Mattson and A. Messac. Pareto frontier based concept selection under uncertainty, with visualization. *Optimization and Engineering*. Invited Paper, Special Issue of Multidisciplinary Design Optimization, IN PRESS.

[5] A. Messac, A. Ismail-Yahaya, and C. A. Mattson. The normalized normal constraint method for generating the Pareto frontier. *Structural and Multidisciplinary Optimization*, 25(2):86–98, 2003.

[6] A. Messac, S. Van Dessel, A. A. Mullur, and A. Maria. Optimization of large-scale rigidified inflatable structures for housing using physical programming. *Structural and Multidisciplinary Optimization*, 25:1–12, 2003.

[7] Vanderplaats R&D. Genesis structural analysis and optimization, 2001. Version 7.0.

[8] L. L. Howell. *Compliant Mechanisms*. John Wiley & Sons, New York, NY, 2001.

[9] C. A. Mattson, L. L. Howell, and S. P. Magleby. Development of commercially viable compliant mechanisms using the pseudo-rigid body model: Case studies of parallel mechanisms. *Journal of Intelligent Material Systems and Structures*. IN PRESS.